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The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics

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Abstract

In 2005, the International Association for the Evaluation of Educational Achievement (IEA), Michigan State University, and the Australian Council for Educational Research took an important step in advancing the field of education by partnering to develop and implement the first international and comparative study of mathematics teacher education. The study was made possible by the substantial funding received from the National Science Foundation, the IEA, and the collaboration of 17 participating countries. The purpose of this article is to illustrate the methodology used in this major cross-national study of teacher education—the IEA Teacher Education and Development Study in Mathematics, known as TEDS-M—and to share its main findings related to the mathematical preparation of future teachers.

Keywords

teacher education/development, mathematics education, international/comparative, methodology

> Teachers make a difference. The success of any plan for improving educational outcomes depends on the teachers who carry it out and thus on the abilities of those attracted to the field and their preparation. Yet there are many questions about how teachers are being prepared and how they ought to be prepared.
>
> —National Research Council (2010, p. 1)

Discussions about teacher preparation are particularly important for future teachers of mathematics because mathematics proficiency has been seen for many years as a requirement for full participation in civil society and a global economy (Commission on Mathematics and Science Education, 2009; Husen, 1967). However, for years, researchers in some countries have reported that mathematics teachers in primary and lower secondary schools often show serious misunderstandings, and these researchers have expressed concern for what is perceived as deficient preparation in this area (e.g., Ball & Bass, 2003; Fennema & Franke, 1992; Post, Harel, Behr, & Lesh, 1991). Teachers may know the facts and procedures that they teach but often have relatively weak understandings of the conceptual basis for that knowledge and have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations (Ball, 1991). Some scholars claim that teachers' knowledge of mathematics, or lack thereof, may help explain the relative performance of students in national or international achievement tests (Darling-Hammond, 2000; Ingersoll, 1999; Kilpatrick, Swafford, & Findell, 2001; Ma, 1999).

Recent research has begun to advance our understanding of the mathematical knowledge considered most important for school mathematics teaching, but we know much less about the knowledge most important for teaching secondary school mathematics than for primary mathematics (see, e.g., Baumert et al., 2010; Hill, Sleep, Lewis, & Ball, 2007; Schmidt et al., 2007). Recommendations from mathematical societies, for example, *The Mathematical Education of Teachers* (Conference Board of Mathematical Sciences, 2001), emphasize that future teachers of school mathematics need to develop a deep understanding of the mathematics they will teach. A recent review commissioned by the National Academy of Sciences in the United States concurs: "Successful mathematics teachers need preparation that covers knowledge of mathematics, of how students learn mathematics,

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and of mathematical pedagogy that is aligned with the recommendations of professional societies" (National Research Council, 2010, p. 123). It also recommends that "both quantitative and qualitative data about the programs of study in mathematics offered and required at teacher preparation institutions is needed, as is research to improve understanding of what sorts of preparation approaches are most effective at developing effective teachers" (p. 124).

As Husen (1967) argued, international comparative studies of education help educators view their own systems of education more objectively because factors potentially related to educational achievement have to be defined in a standardized way. Even and Ball (2009) noted that preparing and maintaining a high-quality, professional teaching force that can teach mathematics effectively is a worldwide challenge and that all researchers can benefit from a worldwide conversation. Research conducted cross-nationally by Britton, Paine, Raizen, and Pimm (2003), Hiebert and colleagues (2003), and Ma (1999) indentified differences in teacher preparation that may explain some differences in school mathematics performance. However, these studies used relatively small or non–randomly chosen samples, so their results are limited in the extent to which they can be generalized.

This article reports on the Teacher Education and Development Study in Mathematics (TEDS-M), a large quantitative comparative study that investigated the mathematics preparation of primary and secondary school teachers in 17 countries: Botswana, Canada, Chile, Chinese Taipei (Taiwan), Georgia, Germany, Malaysia, Norway, Oman, the Philippines, Poland, Russia, Singapore, Spain, Switzerland, Thailand, and the United States. TEDS-M was conducted under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) and was managed by a team of researchers at Michigan State University and the Australian Council for Educational Research.

Rationale and Goals of TEDS-M

Studies such as the Trends in International Mathematics and Science Study (National Center for Education Statistics, 2009) have prompted policy makers in the United States and abroad to call for the development and implementation of rigorous standards for K-12 education, standards as challenging as those of the high-achieving countries. Although higher standards should permeate the whole curriculum, special emphasis has been given to mathematics among other equally important domains of knowledge such as language and science. A key question, however—and at the core of our cross-national study—is whether teacher education in the United States (and in other countries) can manage to prepare future teachers to teach more demanding curricula and standards in mathematics, commensurate to the preparation future teachers receive in the high-achieving countries. Consequently, the overall goal of TEDS-M was to study in a group of countries *how primary and secondary school mathematics teachers learn to teach subject matter content effectively to a wide variety of students* as a result of their preparation programs. This comparative approach to exploring teacher education and its influence cross-nationally helped us to understand the combination of teacher education policies, learning opportunities, and levels of mathematics knowledge that future teachers reach in those countries where pupils show high mathematics achievement vis-à-vis those who do not. The intent of TEDS-M is to replace myths about when, what, and how teachers learn with facts and conclusions backed by rigorous research. Three major research questions were investigated:

1. What intended and implemented policies support the development of prospective primary and lower secondary teachers' knowledge of mathematics as well as related teaching knowledge? How do teacher preparation policies influence the structure of opportunities to learn mathematics for primary and lower secondary teachers at national and institutional levels?
2. What learning opportunities are available to prospective mathematics teachers that allow them to attain such knowledge?
3. What level and depth of mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers enable them to teach the kind of demanding mathematics curricula currently found in the higher achieving countries and required by the higher standards adopted by many states?

Taking up the call by Wilson, Floden, and Ferrini-Mundy (2002) "to make public our research practices" (p. 201), the first part of this article describes key aspects of research methods used by TEDS-M . The second part reports selected findings related to learning opportunities available to prospective mathematics teachers and to the level and depth of mathematics and teaching knowledge they attained at the end of their preservice teacher education. At the end of the article, we discuss how these findings help inform current and future policy.

Because of the limited space allowed for each contribution to this journal and the recent release of the Mathematics Common Core State Standards Initiative[1] in the United States, we limit the presentation of results to opportunities for learning school and tertiary mathematics available to future teachers and to the mathematics content knowledge (MCK) attained by future teachers in the participating countries. Additional results may be found in the TEDS-M international report by Tatto et al. (in press).

Method

Populations and Samples

Definitions. Teacher education is structured differently across national settings and even between institutions in the same country (Tatto et al., 2008, p. 17). Hence, it was important to define key concepts to reflect the national organization of teacher education and also to allow for the design of samples that could be compared across countries.

A teacher education *program* was defined as a set of courses, subjects, and experiences within an institution that leads to the award of a teaching credential upon its completion. A *route* was defined as a set of teacher education programs sharing a number of common features, which can be identified in similar ways in the different participating countries. The *target population of future teachers* comprised all members of a route, in their last year of training, enrolled in an institution offering formal opportunities to learn to teach mathematics, explicitly intended to prepare individuals qualified to teach mathematics in Grades 1 to 8.

Four target populations were studied in TEDS-M: (a) institutions where future primary and secondary teachers are prepared to teach mathematics; (b) teacher educators who prepare future teachers in mathematics, mathematics pedagogy, and general pedagogy; (c) future teachers in their last year of training, preparing to teach at the primary level (*International Standard Classification of Education* [ISCED] Level 1—primary or basic education, Cycle 1); and (d) future teachers in their last year of training, preparing to teach at the secondary level (ISCED Level 2—lower secondary or basic education, Cycle 2; United Nations Educational, Scientific and Cultural Organization, 2007).

However, in some countries, distinguishing between primary and lower secondary levels was not feasible. For example, in Chile the largest program type prepares generalist teachers for Grades 1 to 9. In such situations, a sampling plan was devised to assign half the future teachers randomly to respond to the survey for primary education, whereas the other half was selected to respond to the survey for lower secondary education.

Sample requirements. The international sampling plan used a stratified, multistage probability sampling design. To allow for reliable estimation and modeling, the minimum sample size was set at 50 institutions or programs per route and level, and the effective sample size was set at 400 future teachers per route and level in a given country. *Effective sample size* means that the sample design must be as efficient (i.e., precise) as a simple random sample of 400 teachers from a (hypothetical) list of all eligible future teachers found in a level and route.

However, not all countries prepared enough future teachers to implement these requirements. The actual number of future teachers required for each level and route within the selected institutions and overall was dictated mainly by the total number of institutions in the country, the size of the institutions in the country, and the selection method used in the institutions. Nevertheless, national sampling plans were designed so that each individual would have the same final estimation weight.

In practice, each national sampling plan was unique. A stratified multistage probability sampling plan was used in four countries (Philippines, Russian Federation, Spain, and the United States). Censuses of institutions, educators, and future teachers were taken in six countries (Botswana, Georgia, Norway, Oman, Singapore, and Thailand). Some combination of censuses and samples was used in the remaining countries (Canada, Chile, Chinese Taipei, Germany, Malaysia, Poland, and Switzerland). More details about sampling can be found in the TEDS-M technical report (Tatto, in press).

Ultimately, TEDS-M surveyed 15,163 future primary teachers; 9,389 future secondary teachers; 500 teacher preparation institutions, including 451 preparing future primary teachers and 339 preparing future secondary teachers; and 4,837 teacher educators.

Participation Rates and Adjudication

The IEA's quality standards required minimum participation rates for all target populations in order to ensure that bias resulting from nonresponse was kept within acceptable limits. For each country, participation rates were calculated and reported separately for the four different target populations. The minimum requirement for calculation of key statistics for international comparisons was an overall (combined) participation rate (weighted or unweighted) of at least 75% or that the participation rate (weighted or unweighted) of institutions for the considered population and the participation rate for individuals within the participating institutions were at least 85%.

For each country and for each source of data, a judgment was made about the extent to which the IEA sampling standards had been met. After all samples were adjudicated, recommendations were made for data reporting. For samples with low participation rates, special annotations were written for tables or figures, or in extreme cases, data were excluded from reporting. Participation rates for the surveys conducted in Canadian provinces were so low that no data about Canada can be reported with confidence. For other details related to populations and samples, see the TEDS-M technical report (Tatto, in press).

Instruments

The questionnaires for both primary and secondary future teachers included a common set of questions about their backgrounds; opportunities to learn; and beliefs about mathematics, teaching, and learning. In addition, each questionnaire had items to assess mathematics knowledge for teaching (defined as mathematics content knowledge and mathematics pedagogy knowledge) appropriate for that level. Both questionnaires were developed in consultation with cross-national teams of mathematicians, mathematics educators, psychometricians, and comparative research design experts. Items were written and reviewed by researchers from TEDS-M countries as well as from countries that did not participate in TEDS-M. Members of the TEDS-M management team wrote additional items. All items were piloted in five countries in 2006. A major field trial of instruments and procedures was conducted in 2007.

Measuring Future Teachers' Opportunities to Learn

As Floden (2002) noted, *opportunity to learn* (OTL), has been defined in many ways since the term was first introduced by Husen (1967). For instance, *OTL* may refer to aspects of curricula, instructional materials, instructional experiences, or time available for instruction. Including measures of OTL in TEDS-M allows researchers to describe curricular variation among teacher preparation program types across countries and to investigate how such variation is related to differences in levels of knowledge of future teachers.

Previous research by Tatto, Nielsen, Cummings, Kularatna, and Dharmadasa (1993), Tatto (1996), and Schmidt et al. (2007) was used to develop OTL indices. The connections from prior research and theory provide strong validity-related evidence regarding the content, meaningfulness, and appropriateness of the OTL scales. Expert judgment and techniques such as exploratory and confirmatory factor analysis, scale reliability analyses, and Rasch scaling were used to select the items.

The OTL items were grouped into seven scales: (1) university- or tertiary-level mathematics, (2) school-level mathematics, (3) mathematics education/pedagogy, (4) education/pedagogy, (5) classroom diversity and reflection on practice, (6) school experience and the practicum, and (7) coherence of the teacher education program. The first two address opportunities to learn mathematics; the other five cover other aspects of teacher education. In this article, we report only on opportunities to learn tertiary and school level mathematics.

The items about tertiary-level OTL asked whether future teachers had at any time studied four key mathematics areas: (1) continuity and functions (e.g., beginning calculus, calculus, multivariate calculus, advanced calculus or real analysis, and differential equations), (2) discrete structures and logic (e.g., linear algebra, set theory, abstract algebra, number theory discrete mathematics, and mathematical logic), (3) geometry (e.g., foundations of geometry or axiomatic geometry, analytic or coordinate geometry, non-Euclidean geometry, and differential geometry), and (4) probability and statistics (e.g., probability and theoretical or applied statistics). Because opportunities to learn in some of these areas may occur before or during future teachers' preservice education (e.g., analytic geometry may be studied in secondary school or in tertiary education within or outside teacher education programs), questions asked future teachers whether they had ever studied or not in those areas.

The items about OTL in the school-level domains asked future teachers if during their teacher preparation programs they had studied seven school mathematics areas: (1) numbers (e.g., whole numbers, fractions, decimals, integers, rational, and real numbers; number concepts; number theory; estimation; ratio and proportionality); (2) measurement (e.g., measurement units; computations and properties of length, perimeter, area, and volume; estimation and error); (3) geometry (e.g., one-dimensional and two-dimensional coordinate geometry, Euclidean geometry, transformational geometry, congruence and similarity, constructions with straightedge and compass, three-dimensional geometry, and vector geometry); (4) functions, relations, and equations (e.g., algebra, trigonometry, and analytic geometry); (5) data representation, probability, and statistics; (6) calculus (e.g., infinite processes, change, differentiation, and integration); and (7) validation, structuring, and abstracting (e.g., Boolean algebra, mathematical induction, logical connectives, sets, groups, fields, linear space, isomorphism, and homomorphism).

Items from tertiary and school level mathematics were developed into scales. In this article, we report the proportion to which areas in those domains were covered, according to future teachers, and also the proportion of future teachers who reported covering each individual area (or item) of the domain.

Measuring Future Teachers' Mathematical Knowledge for Teaching

Test design. MCK for both primary and secondary future teachers was assessed by items that spanned four domains, including (1) number (whole numbers, fractions, decimals, integers; number sentences, patterns and relationships, ratios, proportions and percents, and number theory), (2) geometry (geometric shapes, geometric measurement, location, and movement), (3) algebra (patterns, algebraic expressions, equations, inequalities, formulas, and functions), and (4) data (data organization and representation, data reading and interpretation, and chance). In addition, the items included questions about more advanced topics in each domain (e.g., irrational, real, and complex numbers, and topics from calculus, analysis, linear algebra, and abstract algebra in the algebra domain). Each MCK item was further classified by a cognitive subdomain: knowing (recall, recognize, compute, and measure), applying (classify, order, represent, model, and solve routine problems), or reasoning (analyze, generalize, synthesize, integrate, justify, prove, and solve nonroutine problems), and by curricular level (novice, intermediate, or advanced).

Test items were arranged in blocks, one set of five blocks for primary mathematics and another set of three blocks for lower secondary mathematics. To avoid placing too great a burden on each participant, the blocks were rotated among test booklets so that each future teacher answered two blocks of items. The number of future teachers available to take the test was directly related to the number of blocks; in every country there were more future primary teachers than future lower secondary teachers in the TEDS-M samples.

The test design for the primary and secondary booklets allowed analysis of the full covariance matrix and provided enough items and score points to generate scales. Overall, across both the primary and secondary blocks, about 30% of the items were allocated each to number, algebra, and geometry, and 10% to data. About 70% of the questions were multiple choice or complex multiple choice; about 30% were open constructed response. Most of the latter were scored to allow for partial credit when warranted.

Scale Development

Item response theory (IRT) was used to create reporting score scales that would allow estimates of knowledge performance for each person in the study (see, e.g., De Ayala, 2009). The process included calibration of the test items to determine if the data had a good fit with the IRT models. Items with poor fits were reviewed (e.g., combining score categories on items with multiple score points) or removed from the computation of the reported scores. The resulting sets of items were calibrated again, using weights, so that each country contributed equally to the calibration.[2] The final calibration results were used to estimate the location of the examinees on a common IRT scale and were then transformed so that the international mean for the calibration sample on each of the MCK (and the mathematics pedagogy knowledge) scales was 500 and the international standard deviation was 100.

Developing Anchor Points

Anchor points are specific values on each score scale that are used to develop descriptions of what examinees at or near that point on the scale know and can do. To develop the descriptions of the skills and knowledge at a given anchor point, two sets of test items were identified for each anchor point. The first set of test items were those that the person at the anchor point on the score scale was projected to be able to answer correctly, with a probability of .70 or greater, from the IRT model. The value .70 was selected because it is often used as a definition of high-ability test items in item mapping and standard setting. The other set of items selected for each anchor point were those items for which the persons at the anchor point were projected to have a probability of .50 or less of responding correctly, again based on the IRT model. The value .50 was selected because it is often used as a definition of low-ability test items in item mapping and standard setting.

For each anchor point, committees of expert mathematicians and mathematics educators, who participated in workshops held specifically for that purpose at the TEDS-M International Study Center at Michigan State University, analyzed sets of items and developed descriptions of the capabilities of persons near that point. The resulting anchor point descriptions gave tangible meanings to points on the reporting score scales.

Our results include estimates of the proportion of future teachers in the sample from each country who reached or exceeded each anchor point. Hence, for the entire cohort of future teachers in the sampled target population, we have empirical evidence of performance levels (e.g., with probabilities greater than .70 or less than .50 or between .50 and .70) based on what they were projected, from the IRT model, to be able to do or not do within the specified probabilities.

Study Findings

This article reports on the opportunities future teachers had to learn *tertiary* and *school level mathematics* and their performance on the tests of MCK. Because of differences across teacher education programs within countries, whole-country comparisons are not the purpose of TEDS-M. Rather, TEDS-M results compare programs cross-nationally according to the intended grade level and specialization in mathematics of the teachers the countries expect to prepare—teachers who are being prepared to undertake similar roles once they are qualified. Among those who qualify to become primary teachers, most will become generalist teachers, which, depending on the country, may be no higher than Grade 4 or through Grade 6. In a few countries, generalist teachers are prepared to teach either primary or lower secondary grades up through Grade 10. Other future primary teachers qualify to become specialist teachers of mathematics. In contrast, most future teachers of lower secondary school are prepared as mathematics specialists. Some are qualified to teach only up to Grade 8, whereas others are qualified to teach to Grade 12 and beyond. Thus, the findings of future teachers who answered the primary surveys are presented for the four program groups:

1. Lower primary generalists (Grade 4 maximum)
2. Primary generalists (Grade 6 maximum)
3. Primary/lower secondary generalists (Grade 10 maximum)
4. Primary mathematics specialists

Table 1. Primary Future Teachers' Opportunities to Learn Mathematics by Domain

				Tertiary-level domains[1]		School-level domains[2]	
Program group	Country	*n* (tertiary level)	*n* (school level)	Mean percent covered	Standard error	Mean percent covered	Standard error
1. Lower primary (Grade 4 maximum)	Georgia	478	502	0.52	0.01	0.64	0.01
	Germany	918	926	0.23	0.01	0.37	0.01
	Poland[a]	1,797	1,809	0.45	0.00	0.44	0.01
	Russian Federation[b]	2,244	2,260	0.55	0.01	0.74	0.01
	Switzerland[c]	121	121	0.54	0.01	0.49	0.02
2. Primary (Grade 6 maximum)	Chinese Taipei	923	923	0.50	0.01	0.64	0.01
	Philippines	589	591	0.62	0.02	0.75	0.02
	Singapore	261	263	0.38	0.02	0.62	0.01
	Spain	1,092	1,093	0.55	0.01	0.68	0.01
	Switzerland[c]	813	813	0.60	0.01	0.49	0.01
	United States[d]	1,289	1,290	0.42	0.01	0.69	0.01
3. Primary/ secondary (Grade 10 maximum)	Botswana[e]	83	86	0.46	0.02	0.72	0.01
	Chile[f]	649	657	0.43	0.01	0.59	0.01
	Norway (ALU)[g]	392	392	0.47	0.01	0.75	0.01
	Norway (ALU+)[g]	159	159	0.59	0.01	0.83	0.01
4. Primary (mathematics specialists)	Germany	97	97	0.48	0.03	0.62	0.03
	Malaysia	570	571	0.71	0.01	0.72	0.01
	Poland[a]	300	300	0.88	0.01	0.93	0.01
	Singapore	117	117	0.38	0.03	0.62	0.02
	Thailand	658	659	0.85	0.00	0.92	0.01
	United States[d]	187	187	0.48	0.02	0.72	0.01

Adapted from source: *TEDS-M International Report, Chapter 7* (Tatto, Schwille, Senk, Bankov, Rodriguez, Reckase, Ingvarson, Rowley, & Peck, in press). Samples' limitations are listed in the appendix and are indicated in the table by superscript letters.
1. Tertiary-level domains are geometry, discrete structures and logic, continuity and functions, and probability and statistics.
2. School-level domains are numbers, measurement, geometry, algebra and functions, probability and statistics, calculus, and structure.

The findings from future teachers who answered the secondary surveys are presented for two additional program groups:

5. Lower secondary (Grade 10 maximum)
6. Lower and upper secondary (Grade 11 and above)

Opportunities to Learn Mathematics Among Future Primary Teachers

Table 1 shows the average proportion of areas studied by future primary teachers in the domains of tertiary-level mathematics and school-level mathematics. Opportunities to learn in the tertiary-level domain range widely within and between program groups. For example, in Group 1 the lowest average coverage was reported by future lower primary generalist teachers in Germany, and the highest in the Russian Federation. Among future primary specialist teachers, those from Germany again reported the lowest coverage, and mathematics specialists in Poland reported having the highest tertiary-level opportunities to learn. Overall, about half the program types reported mean coverage of 50% or more tertiary domains, and about half reported coverage of less than half; in contrast and also in Table 1, future teachers in most primary program groups reported covering an average of at least 60% of the domains classified as belonging to school-level mathematics.

A more detailed examination explored the percentage of teachers who reported covering individual areas in the domain (not shown in a table due to space restrictions). Among the tertiary-level domains, for instance, more than 80% of the future teachers in the TEDS-M primary samples reported studying number theory, and more than 70% reported studying probability. At least 60% of future primary teachers in most countries reported covering calculus. However, lower proportions studied calculus in Group 1 in Germany; in Group 2 in the Philippines, Singapore, and the United States; in Group 3 in Chile and Norway (ALU), and in Group 4 in Singapore and the United States. More than 70% of future primary teachers in some countries in each program group also reported studying linear algebra, for example, Poland and Switzerland in Group 1; Chinese Taipei, the Philippines, and Switzerland in Group 2; Botswana and Norway (ALU+) in Group 3; and Malaysia, Poland, and Thailand in Group 4.

For the school-level domains, a high proportion of future primary teachers across countries and program groups reported coverage of numbers and measurement, whereas geometry coverage varied widely. However, close to 100% of future teachers in programs training mathematics specialists reported studying geometry in Germany, Norway, Poland, Singapore, Thailand, and the United States; this is also true of teachers being prepared to teach up to Grade 4 and Grade 6 in Russia, Singapore, Spain, and the United States. In contrast, about 50% of future primary teachers prepared to teach up to Grade 4 reported having opportunities to learn geometry in their teacher education programs in Germany, Poland, and Switzerland. Opportunities to learn functions, probability, calculus, and structure are generally low, with the exception of programs training mathematics specialists in Poland and Thailand. In general, as the education of primary future teachers shifts toward the higher grades and becomes more specialized, an emphasis on the areas of functions, data, calculus, and structure becomes more prominent.

Opportunities to Learn Mathematics Among Future Secondary Teachers

Table 2 shows coverage in the domains of tertiary-level mathematics and school-level mathematics for future secondary teachers. Opportunities to learn tertiary-level mathematics vary within and across program groups, with future teachers preparing to teach up to Grade 11 or beyond generally covering a higher proportion of domains than those preparing to teach to Grade 10. In Group 5, future secondary teachers report covering above 70% of the tertiary mathematics domains in Poland, the Philippines, and Switzerland. The lowest proportion of tertiary mathematics coverage was reported in Group 5 by future teachers in Chile, Singapore, and the United States (about 40%). In Group 6, with the exception of Singapore and Norway (PPU), future teachers in all other programs reported covering at least 70% of areas in this domain, whereas the Russian Federation and Chinese Taipei reported covering 90% or above of the tertiary domains. In contrast, Table 2 also shows that future secondary teachers in both program groups generally have substantial opportunities to study school-level mathematics, with an average coverage of 70% or above reported by all except Chile and Germany in Group 5.

A more detailed examination explored the percentage of teachers who reported covering individual areas in the domain (not shown in a table due to space restrictions). Among the tertiary-level domains, little variation was reported in coverage of number theory and probability, with more than 80% of future teachers in Groups 5 and 6 reporting opportunities to study these areas. Considerable variation was reported in opportunities to learn various levels of calculus. Among Group 5, for example, close to 100% of future teachers in Poland reported studying beginning calculus and calculus, whereas in Chile, Norway, and the United States not more than 55% reported studying these areas. In Group 6, at least 95% of future teachers in all program types reported studying calculus, and not surprisingly, much higher proportions of future lower secondary teachers in Chinese Taipei and Poland also reported studying multivariate calculus than in all other countries, with the lowest opportunities to learn reported by future teachers in Chile, Norway, and the United States. In Group 5, coverage of foundations of geometry, discrete mathematics, and differential equations was generally reported to be highest by future teachers in Poland; whereas future teachers from Chile, Norway, and the United States reported the lowest opportunities to learn such topics. Across both program Groups 5 and 6, there were fewer opportunities to learn discrete mathematics than other tertiary domains. Among Group 5, future teachers in Switzerland reported the greatest coverage, and those in Chile and the United States, the lowest (below 20%). Among future teachers in Group 6, those in Chinese Taipei and the Russian Federation reported the highest coverage of discrete mathematics, 98% or higher, followed by Poland and the United States, with the rest close to 50% or below.

In the school mathematics domains, a detailed examination of the responses of future secondary teachers revealed that generally there is high coverage of number, measurement, and geometry across all program groups and countries. However, there are striking differences within and between program groups in the domain composed of more advanced topics such as functions, probability, calculus, and structure. Future secondary teachers in Chinese Taipei, Malaysia, Poland, the Russian Federation, and Thailand reported the highest opportunities to learn in this area, and Chile and Germany reported the least. Differences also exist between some program types within countries. For example, respondents preparing to teach up through Grade 10 in Germany, Singapore, and the United States had fewer opportunities to study areas in this domain than those preparing to teach beyond Grade 10 in their countries. However, in Botswana and Poland no differences were reported between program groups.

Mathematics Content Knowledge of Future Primary Teachers

This section has two parts. The first gives qualitative descriptions developed by expert panels for the primary MCK anchor points. The second part describes the results by program group and country in relation to the anchor points (see Table 3). This allows a comparison of performance levels across the different programs studied.

Table 2. Secondary Future Teachers' Opportunities to Learn Mathematics by Domain

Program group	Country	*n* (tertiary level)	*n* (school level)	Tertiary-level domains[1]		School-level domains[2]	
				Mean percentage covered	Standard error	Mean percentage covered	Standard error
5. Lower secondary (Grade 10 maximum)	Botswana[a]	34	34	0.59	0.03	0.79	0.02
	Chile[b]	733	745	0.44	0.01	0.59	0.01
	Germany	405	400	0.47	0.01	0.60	0.01
	Philippines	731	731	0.71	0.01	0.81	0.01
	Poland[c]	158	158	0.84	0.01	0.94	0.01
	Singapore	140	141	0.40	0.02	0.72	0.02
	Switzerland[d]	141	141	0.71	0.01	0.79	0.02
	Norway (ALU +)[e]	150	151	0.56	0.01	0.82	0.01
	Norway (ALU)[e]	352	355	0.46	0.01	0.75	0.01
	United States[f]	169	169	0.42	0.02	0.71	0.03
6. Lower/upper secondary (Grade 11 or above)	Botswana	19	19	0.72	0.02	0.77	0.03
	Chinese Taipei	365	365	0.90	0.00	0.89	0.01
	Georgia[g]	75	77	0.80	0.02	0.77	0.02
	Germany	359	348	0.71	0.01	0.71	0.01
	Malaysia	388	388	0.78	0.01	0.91	0.01
	Oman	176	268	0.86	0.01	0.87	0.01
	Poland[c]	140	140	0.92	0.01	0.91	0.02
	Russian Federation[h]	2,133	2,135	0.95	0.00	0.92	0.01
	Singapore	250	250	0.63	0.01	0.81	0.01
	Thailand	651	650	0.85	0.00	0.92	0.01
	Norway (Masters)[e]	22	22	0.68	0.04	0.84	0.04
	Norway (PPU)	43	43	0.64	0.02	0.80	0.02
	United States[f]	434	434	0.77	0.01	0.80	0.02

Adapted from source: *TEDS-M International Report, Chapter* 7 (Tatto et al., in press). Samples' limitations are listed in the appendix and are indicated in the table by superscript letters.
1. Tertiary-level domains are geometry, discrete structures and logic, continuity and functions, and probability and statistics.
2. School-level domains are numbers, measurement, geometry, algebra and functions, probability and statistics, calculus, and structure.

Description of Primary-Level Anchor Points

Primary MCK Anchor Point 1. Future teachers of primary school mathematics who answered the TEDS-M test and achieved Anchor Point 1 were generally successful performing basic computations with whole numbers, understood the properties of operations with whole numbers, and were able to reason about related concepts such as odd or even numbers. They were able to solve some problems with fractions. Future teachers at this anchor point were successful at visualizing and interpreting standard two-dimensional and three-dimensional geometric figures and could solve simple problems about perimeter. They could also understand straightforward uses of variables and the concept of equivalence and could solve problems involving simple expressions and equations.

Although future teachers at Anchor Point 1 were able to apply whole number arithmetic in simple problem-solving situations, they tended to overgeneralize and had difficulty solving abstract problems and those requiring multiple steps. They had limited understanding of the concepts of the least common multiple and the number line. Their knowledge of proportionality and multiplicative reasoning was weak. They had difficulty solving problems that involved coordinates and problems about relations between geometric figures. Future teachers at this anchor point could make simple deductions, but they had difficulty reasoning through multiple statements and relationships among several mathematical concepts (e.g., such as determining whether subtraction of whole numbers is associative, understanding that there are an infinite number of decimal numbers between two given numbers, finding the area of a triangle drawn on a grid, and identifying an algebraic representation of a numerical relationship between three consecutive even numbers).

Primary MCK Anchor Point 2. Future primary teachers whose responses to the TEDS-M test placed them at Anchor Point 2 could complete the mathematical tasks at Anchor Point 1 successfully. In addition, this group was more successful than future teachers at Anchor Point 1 at using fractions to solve story problems and at recognizing examples of rational and irrational numbers. They knew how to find the least common multiple of two numbers in a familiar context and could recognize that some arguments about whole numbers are logically weak. They were able to determine areas and perimeters of simple figures and had some notion of class inclusion among

Table 3. Descriptive Statistics for Mathematics Content Knowledge, by Program Group (future teachers, primary)

Program group	Country	*n*	*M*	*SE*	*SD*	% missing	Reached Anchor Point 1 %	Reached Anchor Point 1 *SE*	Reached Anchor Point 2 %	Reached Anchor Point 2 *SE*
1. Lower primary (Grade 4 maximum)	Georgia	506	345	3.9	85.3	0.0	11.9	1.4	0.9	0.5
	Germany	907	501	2.9	82.0	2.4	86.4	1.3	43.9	2.1
	Poland[a]	1,799	456	2.3	67.3	0.9	67.9	1.3	16.8	1.2
	Russian Federation[b]	2,260	536	9.9	91.1	0.2	89.7	2.3	57.3	4.6
	Switzerland[c]	121	512	6.4	62.8	0.0	90.5	2.7	44.2	5.4
2. Primary (Grade 6 maximum)	Chinese Taipei	923	623	4.2	84.2	0.0	99.4	0.3	93.2	1.4
	Philippines	592	440	7.6	51.7	0.0	60.7	5.1	6.3	0.9
	Singapore	262	586	3.7	72.4	0.4	100.0	-	82.5	2.3
	Spain	1,093	481	2.6	56.6	0.0	83.4	1.6	26.2	1.6
	Switzerland[c]	815	548	1.9	65.0	0.0	97.2	0.6	70.6	1.4
	Unites States[d]	951	518	4.5	70.0	28.6	92.9	1.2	50.0	3.2
3. Primary/secondary (Grade 10 maximum)	Botswana[e]	86	441	5.9	48.1	0.0	60.6	5.3	7.1	2.8
	Chile[f]	654	413	2.1	64.9	0.4	39.5	1.8	4.0	0.7
	Norway (ALU)[g]	392	509	3.1	69.3	0.0	88.5	1.5	46.9	2.3
	Norway (ALU+)[g]	159	553	4.3	74.0	0.0	96.5	1.4	68.7	3.1
4. Primary (mathematics specialists)	Germany	97	555	7.5	73.9	0.0	96.0	2.1	71.7	7.0
	Malaysia	574	488	1.8	53.5	0.4	88.7	1.1	28.1	1.3
	Poland[a]	300	614	4.8	92.2	0.0	97.9	1.0	91.0	1.6
	Singapore	117	600	7.8	76.1	0.0	98.3	1.2	87.3	2.8
	Thailand	660	528	2.3	75.1	0.0	91.7	0.9	56.2	1.4
	United States[d]	132	520	6.6	63.0	33.2	94.9	1.7	48.1	6.5

Adapted from source: *TEDS-M International Report, Chapter 5* (Tatto et al., in press). Samples' limitations are listed in the appendix and are indicated in the table by superscript letters.

polygons. Future teachers at Anchor Point 2 also had some familiarity with linear expressions and functions.

However, although future primary teachers at Anchor Point 2 could solve some problems involving proportions, they had trouble reasoning about factors, multiples, and percentages. They were unable to solve problems about the area of obtuse-angled triangles involving coordinate geometry. They did not recognize applications of quadratic or exponential functions and had limited success applying algebra to geometric situations, for example, writing a correct statement about the reflection image of the point with coordinates (a, b) over the x-axis, identifying a set of geometric statements that uniquely define a square, and describing properties of the function defined by the ratio of the area and circumference of a circle). Overall, future teachers at Anchor Point 2 generally did well on items testing "knowing" and on standard problems about number, geometry, and algebra, classified as "applying," but they had more difficulty answering problems that require more complex reasoning in applied or nonroutine situations.

Figures 1 and 2 show two items testing MCK from the primary-level survey. The item in Figure 1 tests knowledge of properties of whole numbers. The international percentage correct is for Part A (83%), B (86%), C (92%), and D (60%). Future teachers scoring at Anchor Point 1 in the primary survey were likely to answer Parts A, B, and C correctly with probability of at least 70%, but they had less than a 50% chance of getting Part D correct. That is, future primary teachers at Anchor Point 1 tended to overgeneralize the associative property. In contrast, future teachers scoring at or above Anchor Point 2 had at least a 70% chance of giving the correct response to all four parts of the item in Figure 1. The algebra item shown in Figure 2 was much more difficult for future teachers at both anchor points, with 12% of the international sample earning full credit on this item and an additional 22% receiving partial credit. Even future teachers at Anchor Point 2 had a less than 50% chance of answering this nonroutine item about expressions with variables correctly.

Primary-Level Assessment Results

Table 3 shows the anchor points and the descriptive statistics for the attainment of MCK by future teachers in the primary program groups. Anchor Point 1 represents a lower level of performance and corresponds to a score of 431 on the primary MCK scale; Anchor Point 2 represents a higher level or performance and a scaled score of 516.

Across all program groups and within each participating country, future teachers' scores on the MCK scale varied widely. In each of the four groups the distributions also overlap considerably. That is, even in the lower scoring countries, there are some future teachers who outperformed some future teachers in the higher scoring countries.

Indicate whether each of the following statements is true for the set of all whole numbers a, b, and c greater than zero.

Check one box in each row.

		True	Not True
A.	$a - b = b - a$	$\square_1$	$\square_2$
B.	$a \div b = b \div a$	$\square_1$	$\square_2$
C.	$(a + b) + c = a + (b + c)$	$\square_1$	$\square_2$
D.	$(a - b) - c = a - (b - c)$	$\square_1$	$\square_2$

Figure 1. A complex multiple-choice algebra item from the primary survey. Source: *TEDS-M 2008 Assessment Frameworks* (Tatto, Senk, Bankov, Rodriguez, & Peck, 2011)

Students who had been studying algebra were asked the following question:

For any number n, which is larger, $2n$ or $n + 2$?

Give the answer and show your reasoning or working.

Figure 2. A constructed response algebra item from the primary survey. Source: *TEDS-M 2008 Assessment Frameworks* (Tatto et al., 2011)

There are also differences in performance within all program groups, particularly with respect to the proportion of future teachers who achieved Anchor Point 2. For instance, among the five countries in Group 1, about 90% of future teachers in the Russian Federation and Switzerland reached Anchor Point 1, but only in the Russian Federation did more than half the sample reach Anchor Point 2. In contrast, future teachers in Georgia found the MCK items difficult to answer; less than 12% reached Anchor Point 1 and few reached Anchor Point 2.

Among the countries in Group 2, future teachers in Chinese Taipei and Singapore scored particularly well on the MCK items, with more than 80% scoring reaching Anchor Point 2. Among Group 3, in Chile about 40% and Botswana about 60% of the future teachers reached Anchor Point 1; however, few future teachers in either country achieved Anchor Point 2. Performance in the two Norwegian program types was somewhat higher.

A large majority in each country preparing primary mathematics specialists reached Anchor Point 1, but the proportion of future teachers achieving Anchor Point 2 varied. Future teachers from Poland and Singapore did particularly well, with more than 85% of the respondents having achieved a score at or above Anchor Point 2. Malaysia had the lowest proportion (less than 30%) of future primary specialist teachers reach Anchor Point 2. In most countries that prepare primary future teachers in generalist and specialist programs, the latter typically outperform their generalist colleagues from the same country, with the exception of the United States, where performance of generalists and specialists is remarkably similar (whereas more than 90% reached Anchor Point 1, only about 50% reached Anchor Point 2).

Mathematics Content Knowledge of Future Secondary Teachers

This section has two parts. The first gives qualitative descriptions developed by expert panels for the secondary MCK anchor points. The second part describes the results by program group and country in relation to the anchor points (see Table 4). This allows a comparison of performance levels across the different programs studied.

Secondary-Level Anchor Point Descriptions

Secondary MCK Anchor Point 1. Future teachers of lower secondary school mathematics who reached Anchor Point 1 showed good knowledge of concepts related to whole numbers, integers, and rational numbers and could do computations with them. They could also evaluate algebraic expressions and solve simple linear and quadratic equations, particularly those that are solvable by substitution or trial and error. They were familiar with standard geometric figures in the plane and in space and could identify and apply simple relations in plane geometry. They were also able to interpret and solve more complex problems in number, algebra, and geometry, if the context or the problem type was a commonly taught topic in lower secondary schools.

However, future secondary teachers at Anchor Point 1 had difficulty describing general patterns, solving multistep problems with complex linguistic or mathematical relations, and relating equivalent representations of concepts. They tended to overgeneralize concepts and did not have a good grasp of mathematical reasoning. In particular, they did not consistently recognize faulty arguments or were unable to justify or write proofs.

Secondary MCK Anchor Point 2. Future teachers who achieved Anchor Point 2 performed all of the mathematic problems in Anchor Point 1 successfully. In addition, they seemed to have a more robust notion of function, especially of linear, quadratic, and exponential functions; were better able to read, analyze, and apply abstract definitions and notation; and had a greater ability to make and recognize valid deductive arguments than future lower secondary teachers at Anchor Point 1. They also knew some definitions and theorems from tertiary-level courses and could apply them in straightforward situations.

However, future teachers at Anchor Point 2 were not consistently successful in solving problems stated in purely abstract terms or with problems containing foundational material such as axiomatic systems in geometry. They also made errors in logical reasoning, such as not attending to all conditions of definitions or theorems and confusing the truth of a statement with the validity of an argument, and were

Table 4. Descriptive Statistics for Mathematics Content Knowledge, by Program Group (future teachers, lower secondary)

							Reached Anchor Point 1		Reached Anchor Point 2	
Program group	Country	*n*	*M*	*SE*	*SD*	% Missing	%	*SE*	%	*SE*
5. Lower secondary (Grade 10 maximum)	Botswana[a]	34	436	7.3	37.8	0.0	6.0	4.2	0.0	–
	Chile[b]	741	354	2.5	84.3	0.6	1.2	0.4	0.0	–
	Germany	406	483	4.9	82.9	0.3	53.5	3.4	12.6	2.2
	Philippines	733	442	4.6	49.0	0.0	14.0	3.0	0.2	0.1
	Poland[c]	158	529	4.2	64.8	0.0	75.6	3.5	34.7	3.2
	Singapore	142	544	3.7	48.1	0.0	86.9	3.1	36.6	4.3
	Switzerland[d]	141	531	3.7	50.0	0.0	79.7	3.4	26.7	3.2
	Norway (ALU+)[e]	148	461	4.5	61.9	1.9	36.1	4.0	2.3	0.8
	Norway (ALU)[e]	344	435	3.4	60.9	3.9	19.3	2.0	0.8	0.4
	United States[f]	121	468	3.7	46.4	32.7	33.5	2.2	2.2	1.3
6. Lower & upper secondary (Grade 11 & above)	Botswana[a]	19	449	7.5	39.3	0.0	21.1	7.4	0.0	–
	Chinese Taipei	365	667	3.9	75.2	0.0	98.6	0.8	95.6	1.0
	Georgia[g]	78	424	8.9	84.2	0.0	18.2	4.4	5.0	2.6
	Germany	362	585	4.4	74.7	0.1	93.4	1.5	62.1	2.9
	Malaysia	388	493	2.4	50.8	0.2	57.1	2.3	6.9	0.9
	Oman	268	472	2.4	47.1	0.0	37.1	2.7	1.8	0.6
	Poland[c]	139	549	4.4	65.3	0.8	85.7	2.6	35.7	2.7
	Russian Federation[h]	2,139	594	12.8	96.2	0.1	88.8	1.7	61.2	4.3
	Singapore	251	587	3.8	62.3	0.0	97.6	1.0	62.9	2.6
	Thailand	652	479	1.6	58.6	0.0	41.0	1.5	8.4	1.1
	Norway (PPU & masters)[e]	65	503	7.9	65.6	0.0	57.9	7.9	16.0	5.1
	Unites States[f]	354	553	5.1	57.1	21.3	87.1	2.0	44.5	3.9

Source: *TEDS-M International Report, Chapter 5* (Tatto et al., in press). Samples' limitations are listed in the appendix and are indicated in the table by superscript letters.

unable to recognize valid proofs of more complex statements. Even though they may have been able to make some progress in constructing a mathematical proof, future teachers performing at Anchor Point 2 were not generally successful at completing mathematical proofs (e.g., interpreting standard deviation when distributions are presented visually, working with foundational materials such as axiomatic systems in geometry, writing a complete proof about the sum of two functions) or solving problems about combinations.

Figure 3 shows two items testing future secondary teachers' ability to apply algebra to solve routine story problems. The international percentage correct on Problem 1 was 76%. Future teachers with scores at Anchor Point 1 on the lower secondary MCK scale had a 70% chance of getting this item correct. Notice that in this problem the numbers of marbles held by Peter and James are described as multiples of the number of marbles held by David, so the problem can be solved by setting up a simple linear equation with one unknown and integer coefficients. In contrast, Problem 2 has a more complex linguistic structure, making it less obvious which quantity to use as the base of the comparisons, which, in turn, leads to a somewhat more complex equation. The international percentage correct for Problem 2 was 56%. Future lower secondary teachers with scores at Anchor Point 1 had less than a 50% chance of getting Problem 2 correct. In contrast, future teachers with scores at Anchor Point 2 were likely to get Problem 2 correct with probability at least 70%.[3]

Lower Secondary-Level Assessment Results

Table 4 contains the anchor points and the descriptive statistics corresponding to the attainment of MCK by secondary program groups. Anchor Point 1 represents a lower level of performance and corresponds to a score of 490 on the lower secondary MCK scale; Anchor Point 2 represents a higher level or performance and a scaled score of 559.

Future secondary teachers showed some variation in performance on the MCK items both within and between countries. In general, future teachers in a number of countries in program Group 5 found the items difficult. For instance, less than 10% of future teachers in Botswana and Chile, and less than 40% in the United States and Norway (ALU+), reached Anchor Point 1. Countries that had better-performing future teachers were Singapore, with more than 85% reaching Anchor Point 1, and Poland and Switzerland, with more than 75%. Regarding Anchor Point 2, Singapore had the highest percentage (36%) reaching Anchor Point 2, whereas several other countries had less than 5% reaching this higher level

The following problems appear in a mathematics textbook for <lower secondary school>.

1. [Peter], [David], and [James] play a game with marbles. They have 198 marbles altogether. [Peter] has 6 times as many marbles as [David], and [James] has 2 times as many marbles as [David]. How many marbles does each boy have?

2. Three children [Wendy], [Joyce] and [Gabriela] have 198 zeds altogether. [Wendy] has 6 times as much money as [Joyce], and 3 times as much as [Gabriela]. How many zeds does each child have?

   (a) Solve each problem.

Solution to Problem 1:

Solution to Problem 2:

Figure 3. A constructed response algebra item from the lower secondary survey. Source: *TEDS-M 2008 Assessment Frameworks* (Tatto et al., 2011)

of performance. These countries included Botswana, Chile, Philippines, Norway (ALU+ and ALU), and the United States.

Among countries with programs that could lead to teaching positions in grades higher than 10, future teachers performance was particularly strong in Chinese Taipei and Singapore, with close to 98% of those samples reaching Anchor Point 1; the United States scored at 87%, close to the Russian Federation. More than 95% in Chinese Taipei; 60% of the German, Singapore, and Russian Federation; and 40% of the United State sample scored at or above Anchor Point 2. In contrast, in Botswana and Georgia, where the sample sizes were each quite small, more than 75% of the future teachers prepared for upper secondary levels scored below Anchor Point 1.

Conclusions and Implications

The Teacher Education and Development Study contributed to research on teacher education with both methodological advances and empirically based observations.

Method

Three parts of the TEDS-M research methodology were highlighted in this article: (a) methods developed to select meaningful samples from four populations in disparate teacher education systems; (b) instruments to better understand the teacher education curriculum and specifically the opportunities to learn mathematics given to future teachers; and (c) instruments for assessing the MCK of future primary and lower secondary teachers and descriptions of anchor points that give qualitative descriptions to several points on a Rasch scale of MCK attained by future teachers.

The collaborative work by researchers in many countries has resulted in the development of a common language and definitions that work cross-nationally to reflect the structure and organization of different teacher education systems. The TEDS-M research team showed that it is possible to design sampling plans for teacher education that are sensitive to local conditions and that meet the high technical quality demanded by the IEA. The sampling plans designed and the statistical methods used for weighting and scaling allow meaningful comparisons to be made across participating countries. As documented by Tatto (2001), prior international and U.S. based studies have been thin on measures of teacher knowledge (e.g., in the U.S., Research About Teacher Education [RATE]; Teacher Education and Learning to Teach [TELT]; Investigating Pathways into Teaching in New York City schools [NYC Pathways Study]; TEDS-M strengthens the field in this area.

The descriptions of anchor points allow countries to interpret the knowledge scores of their future teachers in a meaningful way. In addition, anchor points can serve teacher preparation programs around the world to establish benchmarks of performance for their graduates based on the TEDS-M international assessment, which includes some of the highest achieving countries in the world.

Findings

The first, and most surprising, finding from TEDS-M was the variation in structure of teacher education systems. This challenged researchers at every step of the study—designing sampling, analysis, and reporting. Understanding the variation in teacher education allowed the research team to think of alternatives to reporting "league table ranking" of countries' performance. Pursuing this goal, TEDS-M is able to present analysis and report data to show the strengths of teacher preparation systems in different countries relative to future teachers' knowledge, while staying sensitive to local variations in programs' intents (e.g., preparation of future teachers by grade level and degree of specialization).

A second notable finding is the variation in MCK even within program groups. The difference in mean MCK scores between the highest and lowest achieving country in each primary program group was between one and two standard deviations. A difference of two standard deviations is rather substantial, comparable to transforming a score from the 50th to the 96th percentile in a group. In the top-scoring countries within each program group, the majority of future teachers had scores at or above the higher MCK anchor point. Differences between countries within program groups tended to be larger among the secondary groups than among the primary groups. The situation of the United States is notable in that future primary teachers do very well in reaching Anchor Point 1, but only 50% reach Anchor Point 2, whether generalists or specialists; this places the United States below Chinese Taipei, Singapore, and Switzerland in Group 2 (primary generalists) and well below Poland, Singapore, Germany, and Thailand in Group 4 (primary specialists). In the secondary groups, close to 70% of the U.S. teachers do poorly in Group 5 (lower secondary teachers preparing to teach to Grade 10 maximum), which places them substantially below Singapore, Switzerland, Poland, Germany, and Norway in reaching even the lower level of performance (Anchor Point 1). U.S. future teachers, however, do better in the program Group 6 (lower and upper secondary teachers prepared to teach Grade 11 and above) in reaching Anchor Point 1, yet they still score well below the future teachers from Chinese Taipei, Singapore, Germany, and the Russian Federation. In contrast to all of these other countries, with more than 60% of their future teachers reaching Anchor Point 2, more than 55% of U.S. future teachers fail to reach Anchor Point 2. The U.S. future teacher performance is commensurate with the overall learning opportunities (both tertiary and school level) future teachers reported in Tables 1 and 2. Primary future teachers seem to have lower opportunities to learn than their higher achieving counterparts in school mathematics in the areas of geometry, functions, calculus and validation, and abstracting, and in tertiary mathematics in linear algebra, number theory, analytic geometry, beginning calculus, probability, and statistics. Secondary future teachers seem to have lower opportunities to learn school mathematics in data representation, calculus and validation, and abstracting, and tertiary mathematics in linear algebra, analytic geometry, beginning calculus, calculus, probability, and statistics. Thus, a policy recommendation for the United States to be able to prepare teachers at the level of those teachers in countries that show high levels of achievement in international tests would require a more extensive coverage of these areas and domains.

Yet relations between OTL and MCK are complex. Among lower primary generalist teachers, knowledge of mathematics content is strongest in the Russian Federation and opportunities to learn both tertiary and school level mathematics for these future teachers are also the highest in Group 1. However, also in Group 1, future primary generalist teachers in Germany reported the lowest OTL, but they are not the lowest scoring on MCK. That position belongs to future teachers from Georgia. In Group 2, the highest mean MCK performances were achieved by future teachers in Chinese Taipei and Singapore, but they report only moderate opportunities to learn tertiary and school level mathematics topics in their teacher preparation programs. In contrast, future primary generalist teachers in the Philippines reported the highest levels of OTL; however, they had the lowest mean MCK score of countries in Group 2.

OTL and MCK are both potentially mediated by context, program policy, and future teachers' backgrounds. For instance, Singapore actively recruits teachers with high mathematics performance. Therefore, its future teachers in program Group 2 may not need as many opportunities to learn school or tertiary level mathematics as others. Similarly, future teachers in Germany in program Group 4 indicated low levels of OTL related to tertiary- and school-level mathematics. But this may mean that program designers assumed that such content had already been taught earlier.

Mathematics teacher education is also influenced by the system of governance (e.g., whether the state is weak or strong); whether the level of country administrative control is centralized or decentralized; whether programs are held accountable for their performance; and whether the country's philosophy regarding diversity in mathematics knowledge is valued over homogeneity, both within classrooms and among those preparing to become teachers. Thus, in order to understand determinants of MCK, more sophisticated modeling must be undertaken.

Another contribution of TEDS-M is an international database with all data collected by TEDS-M. The database and its documentation provide a shared language and, with the anchor point descriptions, shared benchmarks for examining teacher preparation programs in light of what has proved possible in some contexts. They will soon be available to other researchers for secondary analyses to develop and test their own hypothesis.

Thus, one important message to teacher educators and policy makers is that attention needs to be paid to the emphasis, kind, and depth of the opportunities to learn provided to future teachers. For instance, future primary teachers in high-achieving countries are generally provided with more opportunities to learn both tertiary-level (specifically geometry, continuity, and functions) and school-level (specifically functions, calculus probability and statistics, and structure) mathematics than primary teachers in other countries. This pattern appears to extend to future secondary teachers as well.

Challenges

The findings of this study respond to the latest National Research Council report on teacher preparation that calls for rigorous quantitative studies and begins to answer the urgent questions posed by the National Academies of Science in the United States for preservice teacher education programs, such as who enters these programs, the type of instruction and experiences of these students, and the extent to which the course work and experiences in mathematics is consistent with scientific evidence. TEDS-M shows that it is possible to design sampling plans for teacher education that are sensitive to local conditions and meet high technical quality standards for comparative research. Importantly, the study offers a model for data collection that provides valid, reliable, and cross-national data about the content and pedagogical knowledge of graduates from the various kinds of teacher preparation programs included in the study.

However, as this study progressed, we realized that there is much that we do not know. For instance, TEDS-M does not link the effects of teacher education to pupil achievement. That is, it remains to be seen if the better achieving teachers are also the better classroom teachers. To assess this, much has to be done to develop information systems to allow pursuing this very important question. Indeed, while we collected important data and advanced the methods to study teacher education, we also found ourselves challenged by limited access to usable data within teacher education programs, including the lack of a precise account of the number of students in the program, the hours allocated to each area of study, the qualifications of faculty, a good follow-up system for graduates, and the costs of running the program, among others. Better teacher education program databases are needed to truly develop useful accountability systems.

If the quality of education for every child is to be improved, the education of teachers needs to be taken seriously. In their challenging book, Barber and Mourshed (2007) argue that the world's school systems that "come out on top" do three important things: they get the right people to become teachers, they develop them into effective instructors, and they ensure that the system is able to deliver the best instruction to every child. As Darling-Hammond (2010) has recently argued, "Nations that have steeply improved their students achievement . . . attribute much of their success to their focused investments in teacher preparation and development" (p. 194).

Ultimately, the question at a global level is what it would take to develop competent teachers of mathematics for every child at every level. TEDS-M provides valid, reliable, and rigorous data to inform teacher education policy and to aid in accomplishing this crucial goal.

Appendix

Notes for Primary Tables 1 and 3

AP1 is Anchor Point 1 (431); **AP2** is Anchor Point 2 (516).

a. **Poland**: Reduced coverage: Institutions offering only consecutive programs were not covered. Interpret with caution (combined participation rate between 60% and 75%).
b. **Russian Federation:** Reduced coverage: Secondary pedagogical institutions were excluded.
c. **Switzerland:** Reduced coverage: The population covered includes only institutions where German is the primary language of use and instruction.
d. **United States:** Reduced coverage: public institutions only. Interpret with caution (combined participation rate between 60% and 75%). Data from two institutions were accepted as exceptions because, in each case, one additional participant would have brought the response rate above the 50% threshold. The data contain a number of records that were completed through unapproved testing procedures (using a telephone interview). Of the 1,501 recorded as participants, the full questionnaire was administered to 1,185, of whom 1,083 provided sufficient data to receive scores on this measure. Comparisons with other countries cannot be made with confidence.
e. **Botswana:** The number of respondents is small (86), but the data are reliable, as they came from a census of a small population (100).
f. **Chile:** Interpret with caution (combined participation rate between 60% and 75%).
g. **Norway:** Interpret with caution (combined participation rate between 60% and 75%). Data from one institution were accepted as an exception because one additional participant would have brought the response rate above the 50% threshold. Program types ALU and ALU+ are reported separately because the two populations partially overlap; data from these program types cannot be aggregated.

Notes for Secondary Tables 2 and 4

AP1 is Anchor Point 1 (490); **AP2** is Anchor Point 2 (559).

a. **Botswana:** The number of respondents is small (53), but the data are reliable, as they came from a census of a small population (56).
b. **Chile:** Interpret with caution (combined participation rate between 60% and 75%).
c. **Poland:** Reduced coverage: Institutions only offering consecutive programs were not covered. Interpret with caution (combined participation rate between 60% and 75%).
d. **Switzerland:** Reduced coverage: includes only institutions where German is the primary language of use and instruction.
e. **Norway:** Interpret with utmost caution (combined participation rate 58%). Data from one institution were accepted as an exception because one additional participant would have brought the response rate above the 50% threshold. Program types ALU and ALU+ are reported separately because the two

populations partially overlap; data from these program types cannot be aggregated. Comparisons with other countries cannot be made with confidence.

f. **United States:** Reduced coverage: public institutions only. Interpret with caution (combined participation rate between 60% and 75%). Data from one institution were accepted as an exception because one additional participant would have brought the response rate above the 50% threshold. The data contain a number of records that were completed through unapproved testing procedures (using a telephone interview). Of the 607 recorded as participants, the full questionnaire was administered to 502, of whom 475 provided sufficient data to receive scores on this measure. Comparisons with other countries cannot be made with confidence.

g. **Georgia:** Reduced coverage: The population covered did not include Russian or Azeri sections of institutions. Interpret with caution (combined participation rate between 60% and 75%). Data from two institutions were accepted as exceptions because, in each case, one additional participant would have brought the response rate above the 50% threshold.

h. **Russian Federation:** An unknown number of those surveyed had previously qualified to become primary teachers.

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Notes

1. See the Common Core State Standards Initiative at http://www.corestandards.org/the-standards/mathematics/introduction/how-to-read-the-grade-level-standards/ (retrieved June 5, 2010).
2. For the dichotomous items, the standard Rasch (1980) model was fitted; for polychotomous items, the partial credit model (Masters, 1982) was fitted. Both item types were analyzed simultaneously using ACER Conquest software (Wu, Adams, Wilson, & Haldane, 2007).
3. Examples of item types developed for the knowledge tests and the scoring guides accompanying released items can be found at the TEDS-M official website (http://teds.educ.msu.edu/).
4. Questionnaire items were received from several sources, including study investigators, national research coordinators, and mathematics consultants. Several items were also provided by other studies. TEDS-M has received publication copyright for those items from the following: Copyright 2006, Study of Instructional Improvement (SII) Learning Mathematics for Teaching/Consortium for Policy Research in Education (CPRE), University of Michigan, School of Education, Ann Arbor, MI. Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411. MSU copyright 2006, Developing Subject Matter Knowledge in Math Middle School Teachers (P-TEDS) supported by NSF Grant REC-0231886. Knowing Mathematics for Teacher Algebra (KAT) supported by NSF Grant REC-0337595. TEDS-M investigators also developed their own items with funds provided by a grant from the National Science Foundation Award No. REC -0514431.
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